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RETROSPECT AND GOALS OF TURBULENCE RESEARCH

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ABSTRACT: Review of the high points in the development of the field of turbulence analysis, dwelling in particular on the findings of Rayleigh, Prandtl, and Taylor. Rayleigh's studies of the development of vortex fields in viscous fluids are discussed, the importance of Prandtl's work, including his mixing length hypothesis, is noted, and Taylor's contribution to the development of the statistical theory of turbulence is evaluated. Studies concerning the transition from laminar to turbulent flow, performed by Raleigh and Tollmien, are summarized.

1. INTRODUCTION /975*

Since the end of the last century several generations of scientists have worked in investigation of the turbulence problem. In retrospect it is to be observed that the individual mathematical and experimental investigations have borne the extremely heavy imprint of prematurely formulated hypotheses.

The direction of effort was determined by the creative ideas of several great pioneers in science. Perusal of their original writings reveals comparable stages in their intellectual work, but also brings to light highly individual traits as regards selection of the means employed in their work, so that from both standpoints one can recognize many guiding principles of research strategy. The ideas of O. Reynolds, Lord Rayleigh, L. Prandtl, and G. I. Taylor have exerted a very decisive influence on the work of their successors. Their early conceptions were of vital importance to the rippning of later fruits of scientific endeavor.

But in turbulence research, as indeed in other fields of knowledge, it is to be observed that in the later course of development a tendency arose toward ritualization and canonization. Many cautious early hypotheses were not substantiated by more thorough knowledge or proofs. Rather a state of habituation to ingrained patterns of thought regarding the assumptions was reached,

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*Numbers in the margin indicate pagination in the foreign text.

such that these assumptions had the force of entrenched ideas because of gradual blunting of critical objections to them. Today, however, new views have developed. Progress in the numerical field has made many earlier limitations needless; the possibility is emerging of advancing into hitherto inaccessible areas the investigation of which is of advantage for the mastery of technical processes.

2. THE ROLE OF THE EXPERIMENT

Concern with scientific theory is without exception stimulated by observation of nature and a special controlled form of the latter in an experiment.

In a work of his dating from 1883, Lord Rayleigh describes Kundt's experiments in which not overly fine sand subjected to acoustic excitation in a tube collected at the nodes of air vibrations. He also calls attention to a remark by Faraday that "very fine dust is drawn into the whirlwinds formed above points of vibration if the sand settles above the vibrating points of a vortex or of a plate." (My description, according to John Thyndall in the biography of Faraday.) Rayleigh then demonstrates, through integrals of the equations of motion, that such air vibrations can indeed be verified between two walls.

In the writings of G. Hagen and H. Helmholtz there are already indications that they were acquainted with the difference between the forms of flow which we today designate as laminar and turbulent, and with their effects on energy losses. A first distinguishing characteristic was obtained from the color diffusion experiments of O. Reynolds (1883). A finding published by Prandtl in a paper dating from 1914 may be assessed as an equally great achievement. By means of measurements with a sphere he ascertained that in the case of external flow around a body as well the different kinds of flow motions, laminar and turbulent, are possible, although, as is expressly stated, G. I. Taylor determined the influence of turbulence from his own meteorological measurements, and this merely represents proof of the fact that observation augments our knowledge especially in an early stage of investigation of a phenomenon. It is true that the small differential equations giving an elementary description of all hydrodynamic forms of movement had been known since Navier's lecture at the Parisian Academy of Sciences in 1827, but all references to important types of flow were taken from observation of nature and only subsequently calculated. Human imagination is not capable of divining such phenomena from a differential equation. Observation and experiment continue to play an important role in the process of perception. The fact that immediately at the end of every period of research — which may extend over generations — when the problem has become /976 crystal clear and transparent, all questions can be resolved mathematically, should not lead mathematical specialists to the erroneous conclusion that mere desk work would have served the purposes of their colleagues in their narrow specialty from the early beginning of their endeavors in formulating problems. But an experiment does not merely provide encouragement for theoretical work; in the process of subsequent work it is also a constant companion in the practice of mathematical abstraction. Effects of apparently lesser importance must for the most part be separated from the very manysided natural process and ignored for the sake of simplification in calculation, but even then mathematical simplifications, whose influence on the actual value of the result is difficult to assess, are still needed. A concluding comparative experiment verifying the theory continues to be of the greatest importance. Yet the method of simplifying reasoning also teaches one to adopt the natural process as a guide in approaching new questions for which a decision is to be made by experiment.

3. THE ELEMENTARY TURBULENCE PROCESS

A necessary, but still not sufficient condition for the occurrence of turbulence is the presence of vortices in a viscous fluid. Lord Rayleigh showed that integrals of a differential equation obtained by him from the Navier-Stokes equations are of the vortical type. At the beginning of the calculation he needlessly neglected the non-linear summands. He split the differential equation into an equation of the fourth and an equation of the second order. The former yields two double integrals, which, of course, would also be valid for the complete equation, including the quadratic terms.

Today we are familiar with the complete integral of the unsimplified differential equation for periodic motions in unlimited rotational fields and we observe that in the event of uniform wavelength the non-linear terms vanish from its sum over the entire flow space. This is a criterion for the laminar form of motion. Only in the case of turbulence are the non-linear quantities of a motion periodic in time different from zero (sufficient condition). The argument of the exponential function of each of the four integrals contains the coordinates normal to a smooth wall bounding the fluid. In two integrals the coordinate has the plus sign, and in the other two the minus sign. In order to fulfill the two no-slip conditions, three constants are required on the wall under these conditions: two of them are calculated from an inhomogeneous system, and the third remains undetermined. One of the remaining integrals increases constantly with increasing distance from the wall. Hence the solution is applicable only to a fluid height bounded from above, that is, a quantity of water oscillating over an even surface in gravitational waves. It is to be added that no solution exists for this vortical system, with which the flow medium, bounded on one side by a plane surface, extends to infinity if the motion is to vanish at a great distance from the wall. In a paper published in 1892 Lord Rayleigh described the corresponding vortex motion between two parallel planes with the same integrals. In this case it is calculation of wavelength eigenvalues which is involved.

Although our present-day view is that these studies to describe elementary vortex motions are of particularly great importance, modern turbulence theories being founded on them, Lord Rayleigh's work was not continued until the most recent past.

The work of Ludwig Prandtl, who employed an entirely different approach, was of decisive importance in development of the process of calculation, for advance determination of integral turbulent flow effects. Once when advanced in years he spoke about his method of investigation, reporting that he first of all endeavored to establish a view of the things underlying a problem in order to understand the processes. Not until later did he bring equations into play, when he believed that he had an understanding of the processes, but equations represent a good means of obtaining conclusions and proof which other people as well are inclined to recognize. Prandtl's mode of operation bore the heavy imprint of his study of mechanical engineering -- an engineer tries to arrive at practical conclusions, to obtain knowledge which is ready for application. He was drawn into the Goettingen circle of mathematicians and physicists by the great mathematician Felix Klein, who strives for symbiösis between technical application and scientific theory.

One of the many results arising out of Prandtl's life of work is that all the data on turbulence applied in technology today are without exception based on his findings. The share of turbulence studies in Prandtl's research can be estimated from the number of his publications devoted to this subject. Of 150 printed papers, 19 were devoted to this topic. But what we today are able to read in a few pages of modern textbooks as the distilled essence of his activities was accomplished through persistent effort over a period of decades. Ten years after the lecture on his boundary layer theory, there appeared his paper on boundary transition spheres (1914). Another decade passed before the highly important mixing length equation was recorded; according to Prandtl, it was to serve the purpose of hydrodynamic calculation of the distribution of the main flow of turbulent motion under the most widely varying conditions. The molecular kinematic viscosity is replaced by a vortex viscosity of like dimensions, the magnitude of which must be determined by a characteristic length and a velocity. According to Prandtl, the velocity is that of a traverse movement in which the "bundles of fluid" -- that is, "vortices", in our terminology -- pass through a layer at the same velocity. The length coincides in order of magnitude with the diameter of the vortex. Later (1932) Prandtl asserted that the length corresponds to a distance traversed by a particle of fluid before it mixes again with its environment. In the paper dating from 1925 the functional dependence of the length on distance from the wall remains undetermined. Not until eight years later is the logarithmic dependence of the main velocity on wall distance obtained, with a linear formula after integration, and, lastly, the dependence of the friction coefficient on the Reynolds number is found from the velocity profile. The procedure is not, however, purely analytical; the relation between shear stress and Reynolds number had to be determined /977
through measurement. If, for the sake of simplicity of calculation, the flow is split into a main flow and a time-dependent secondary motion, two differential equations are appropriate, one for the equilibrium averaged with respect to time, and the other for the instantaneous equilibrium. Prandtl worked with the

first equation only, obtaining the missing data by way of experiment. But precisely because Prandtl and his pupils never wandered far from stimulating and verifying experiments in their calculations and from summarizing abstractions, their results are of the highest value for the engineer applying them.

It is the time now to make mention of a quite similar train of reasoning of G. I. Taylor. In 1913 G. I. Taylor was engaged as a meteorologist in measuring atmospheric humidity and temperature above the cold water off the shores of Newfoundland. Immediately above the surface of the water the air exhibited a low temperature, but several hundred meters higher it was appreciably warmer. The air masses had been warmed while crossing the American continent, and then had become substantially colder while flowing over the waters of Newfoundland, which were cooled by icebergs. The actual thermal conductivity had to be greatly increased relative to a molecular material constant in order for such a significant effect to have resulted within a short period. Like kinematic viscosity, thermal conductivity has a dimension corresponding to the product of a length times a velocity. In the case of convection of heat in a vertical direction Taylor could assume that the vertical velocity of flow was of decisive importance. He measured this velocity and formed its absolute mean value with respect to time. Then he was also able to determine the length, by dividing the observed thermodiffusion by the mean value of the vertical velocity. Taylor termed this length "mixture length." In a later paper dating from 1958, he placed a limit on his own accomplishment to that of Prandtl, with the remark that he himself had not attempted to relate this length to the turbulent velocity. Prandtl appears not to have known of Taylor's paper of 1913, since there is no reference to it in Prandtl's publications. The common designation "mixture length" and "Mischungsweg" [mixing length] apparently arose from analogous conclusions on Boltzmann's kinetic theory of gases. In publications of Prandtl's dating from 1932 and 1935 it is even stated that the mixing length is something corresponding to the mean free path length of the molecule in the kinetic theory of gases.

It must not be forgotten, however, that Prandtl's mixing length formula, and especially the logarithmic velocity profile, are approximations valid only in restricted fields of application. Prandtl stated this expressly in later writings, but the restrictions not infrequently have been lost in the writings of his successors.

A theorem recorded in 1926 gave rise to many subsequent misinterpretations. Prandtl wrote that "the variations in velocity and accordingly the mixing motion related to them are the cause of the strikingly great "friction forces" of turbulent motion. Strictly speaking it is not "friction" but an exchange of impulse between adjacent flows moving at different velocities."

As a matter of fact, however, the large inertial terms in the differential equations have comparable friction terms corresponding to them, not in the main motion, but in the small vortices. The large velocity gradients in flow subject to rotation lead to a considerable consumption of work of friction per unit volume. The additional inertial values of turbulent flow were designated as "shear stresses," a concept also very vaguely interpreted by all of Prandtl's successors.

G. I. Taylor's later contribution was formulation of the static turbulence theory. According to this theory, all variant values relating to time and space were to be replaced by their statistical mean. Despite further intensive efforts on the part of many theoreticians, however, it must be said that only a few results proved to be applicable by experts. There are at present many attempts to be observed to proceed from the statistical mean values in order to arrive at conclusions regarding the elementary process in space and time. It would be much simpler to go in the opposite direction, employing the elementary flows underlying the modern theories of turbulence in order to calculate comparable mean values.

Under the influence of an integrating approach doubts arose as to whether a turbulent flow process was susceptible of deterministic definition, or as to whether the irregularities of the flow pattern were to be evaluated as random phenomena. Yet it must be borne in mind that all processes in the sphere of Newtonian mechanics, to which turbulent flow motions unquestionably belong, are deterministically describable. A clarification of linguistic usage is needed at this point, however. Much is designated as random which, strictly speaking, does not deserve such a designation. For example, the number at which a roulette wheel comes to a stop, or the side of a rolled die which finally emerges on top, is called a matter of chance. But both are deterministic processes. The uncertainty in advance determination of the result is accounted for simply by the fact that we are unable to regulate and measure with sufficient precision a factor introduced by a movement of the hand, and that the friction coefficients of the wheel and the die are not available as measured quantities. But no one will contest the fact that this so-called random event can be calculated through integration of a differential equation, that the final state of a movement can be determined in advance if the initial conditions and the inertial and friction coefficients are known.

A principle of order determined by the dynamic equilibrium prevails in turbulent flows as well, both in the elementary process and in the mean values. The purpose of theories, however, is not verification of every variant form of motion but the setting up of models of calculation which emphasize the typical properties of turbulent flows.

4. THE TRANSITION FROM LAMINAR TO TURBULENT FLOW /978

The occurrence of turbulence, that is, the formation of vortices and their separation into shortwave and longwave movements, is an elementary process described by the differential equation. Integration over the time and the path coordinates and fulfilment of the adhesion conditions at the boundaries proved to be extremely difficult and necessitated substantial simplifications in the simplifications in the equations.

In the simplest case of unbounded flow, however, the turbulent or undulatory motion can be strictly calculated for any desired Reynolds number. The damping with time is proportional to the viscosity and inversely proportional to the square of the wavelength. Vortices are but very short-lived at a low Reynolds number. At a higher Reynolds number turbulence is initiated by the superimposition of vortices of different wavelengths. The non-linear inertial summands leads to a separation of the motion into further systems of shorter and longer waves analogous to the acoustic summation and differential tones and to the combinatorial oscillations of elastic systems.

It came to be observed that at very low Reynolds numbers no vortices arose in the case of flow around bodies in the region of slüggish flow; the dimensionless resistance coefficient is inversely proportional to the Reynolds number. At higher Reynolds numbers large adherent vortices are formed -- mostly in pairs -- on the leeward side of obstacles, ones which lengthen in the direction of flow as the Reynolds number is further increased and finally float away. The flow of the adherent vortices is laminar. Turbulence then arises in the wake if motions of different frequency are initiated inside the vortices, and also if detached vortices influence one another. Even in the region of formation of laminar vortices the resistance increases more greatly as resistance with linear velocity; full turbulence is characterized by a resistance proportional to the square of the velocity. Both in the wake of bodies and in boundary layers and channels turbulence does not occur suddenly after a critical Reynolds number is exceeded; rather a transitional zone is formed. Repetition of the Reynolds color experiments for pipes with angular inlets (Figure 1) revealed the transitional region in the Reynolds number range of 1900 to 2400.[1] The transition to turbulence in the greatly retarded boundary layers begins with a large laminar detaching vortex which undergoes turbulent decomposition somewhat downstream from the point of detachment, the boundary layer, in turbulent flow, being able again to adhere to the wall.

At a time when only a few results of observation of the occurrence of turbulence were extant, Lord Rayleigh (1887) investigated by means of calculation the stability of the laminar main flow in the vicinity of a wall. A periodic vortical motion was superimposed on the main flow; both this calculation and its continuation by the Goettingen school starting in 1921 had to be carried out with great simplifications. Lord Rayleigh ignored friction and the influence of curvature of a main velocity profile. The differential equation, in which the quadratic terms of the perturbed motion were also deleted as being small, was satisfied in a series of points and on the wall, as well as at a great distance from the wall. Eigenvalues were obtained from a system of homogeneous equations and by setting the denominator determinant to equal zero.

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[1]Measurement by Dipl.-Ing. Hellmund, TU Dresden.

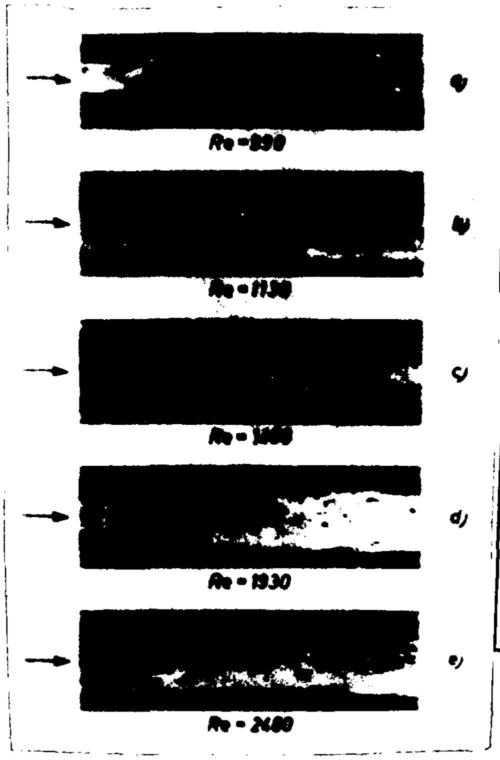

Figure 1. Color Lines As An Indicator of Vortical Movement In A Pipe For the Region of Transition From Laminar To Turbulent Movement. A mathematically rigorous criterion for turbulence is the existence of a convective acceleration vector subject to rotation on superimposition of vortices of different wavelengths. A color line observation is not sufficient for location of the boundary.

After partial results had been achieved by other authors, the final work was accomplished in 1929 by Tollmien. He allowed for friction and curvature in the profile of the main flow, but modified the linearized differential equation in such a way that a fundamental system can be set forth. Pairs of eigenvalues of the wavelength and Reynolds number are calculated for assumed harmonic oscillations from a homogeneous system of equations to satisfy the adhesion conditions and a junction condition at the edge of the boundary layer.

Tollmien's calculations are often subjected to criticism. As early as 1926 Noether adduced proof that no neutral or excited oscillations could exist in the region of large Reynolds numbers with the differential equation which was taken as the basis. G. I. Taylor (1938) expressed doubts as to the correctness of this method. The very fact that the laminar-turbulent transition on bodies has been shown by /979 experiment to depend on the degree of turbulence of the outer flow, and thus on forced vortical fields, is an indication that there occurs in this case a process which must be described by inhomogeneous equations, so that no eigenvalues exist. Both calculations were later associated with a special region, and Tollmien's calculations only with the smallest degrees of turbulence up to vortical velocities of the outer flow lower than two-tenths of a percent of the main velocity.

The results of the theory, particularly the dependence of the critical Reynolds number on the Hagen number characterizing the state of acceleration, were of the highest value in designing low-drag wing profiles with a laminar boundary layer.

The method followed by Tollmien of obtaining a picture of the differential equation integral through approximations is only one of the very many possible methods. Yet in forty years no one has made a radically different formulation for integration of the equation. Nor should the fact be overlooked that all these efforts were initiated by a stability hypothesis of laminar boundary layers advanced by Lord Rayleigh. The idea of regarding disappearance of time-dependent damping as the beginning of turbulence can today be assessed as a necessary condition. Although turbulence is initiated by the superimposition of perturbed motions of different wavelengths, the existence of these perturbed motions for a sufficiently long period is assured only if the motion does not rapidly die out again as a result of damping.

Theoretically, however, it would also be possible to approach the solution to the problem, that is, find a criterion for the laminar-turbulent transition, by proceeding from another point of departure.

This could be accomplished, for example, by proceeding from the recently acquired finding that there are minimum Reynolds numbers formed with the vortical volume and vortical velocity, below which the flow of vortices is always laminar. This relationship was established through evaluation of observations of the period of extinction of individual vortices. Older data when subjected to new evaluation yielded the same result for free jets, reciprocal depressions, and jet boundaries. The Reynolds number at which laminar vortices become turbulent and separate from obstacles is of similar order of magnitude ($Re_{min} \approx 30$ to 50).

If this idea is applied to verify the measured values of the Reynolds number of the boundary layer transition on a smooth plate under different degrees of turbulence (Figure 2), then also all the measured points, without exception, are situated within a family of curves, with a parameter formed from the product of the smallest Reynolds number of the vortex times the ratio of the boundary layer thickness δ to the wavelength λ of the vortex. With degrees of turbulence greater than 1%, all the measured points even fit on the same curve (Figure 2). The smallest Reynolds number is possibly the one at which perturbed movements are rapidly extinguished in a vortex, before they are able to react with adjoining vortices and separate the movement in the manner described into other vortices of a different frequency. But it should be possible to verify the occurrence of turbulence on such a basis only with a differential equation which takes nonlinear addends of the vortical movement into account as well as the change in the main movement with time.

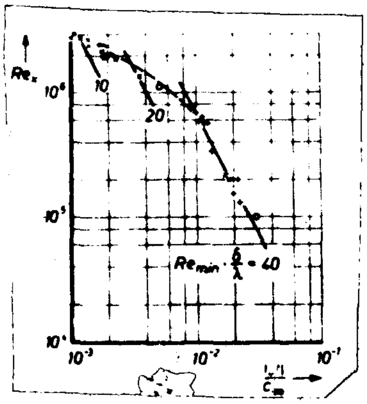

Figure 2. Reynolds number Re_x of Laminar-Turbulent Transition on a Plate Versus Degree of Turbulence $|C'|/C_\infty$:

$$Re_x = \frac{1}{25}\left(Re_{min} \cdot \frac{\delta}{\lambda}\right)^2 \cdot \frac{1}{(|C'|/C_\infty)^2};$$

Measurements according to Schubauer and Skramstad, Dryden, Hall and Hislop, and Baily and Wright.

A statement of Lord Rayleigh's, which he placed at the beginning of his paper of 1883 (already mentioned) was a farsighted one pointing out the direction to be taken by future research. He said: "And even when we are prepared to include in our investigations the influence of friction, by which the motion of fluid in the neighborhood of solid bodies may be greatly modified, we have no chance of reaching an explanation, if, as usual, we limit ourselves to the supposition of infinitely small motions and neglect the squares and higher powers of the mathematical symbols by which it is expressed."

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